

Estimating ε'/ε in the Standard Model

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Abstract

I discuss the comparison of the current theoretical calculations of ε'/ε with the experimental data. Lacking reliable “first principle” calculations, phenomenological approaches may help in understanding correlations among different contributions and available experimental data. In particular, in the chiral quark model approach the same dynamics which underlies the $\Delta I = 1/2$ selection rule in kaon decays appears to enhance the $K \rightarrow \pi\pi$ matrix element of the Q_6 gluonic penguin, thus driving ε'/ε in the range of the recent experimental measurements.

The results announced by the KTeV Collaboration last February and by the NA48 Collaboration at this conference [1] (albeit preliminary) have marked a great experimental achievement, establishing 35 years after the discovery of CP violation in the neutral kaon system the existence of a much smaller violation acting directly in the decays.

While the Standard Model (SM) of strong and electroweak interactions provides an economical and elegant understanding of the presence of indirect (ε) and direct (ε') CP violation in term of a single phase, the detailed calculation of the size of these effects implies mastering strong interactions at a scale where perturbative methods break down. In addition, CP violation in $K \rightarrow \pi\pi$ decays is the result of a destructive interference between two sets of contributions (for a suggestive picture of the gluonic and electroweak penguin diagrams see the talk by Buras at this conference [2]), thus potentially inflating up to an order of magnitude the uncertainties on the individual hadronic matrix elements of the effective four-quark operators.

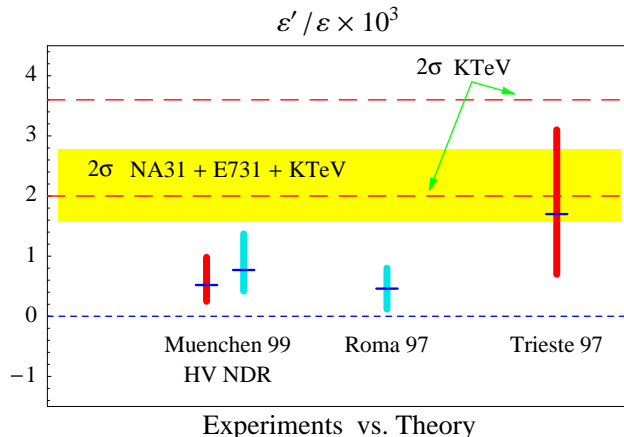


Figure 1: The recent KTeV result (2σ) is shown by the area enclosed by the long-dashed lines. The combined 2σ average of the KTeV, NA31 and E731 results is shown by the gray band. The predicted München, Roma and Trieste theoretical ranges for ε'/ε are shown by the vertical bars with their central values.

In Fig. 1, taken from Ref. [3], the comparison of the theoretical predictions and the experimental results available before the Kaon 99 conference is summarized. The gray horizontal band shows the two-sigma experimental range obtained averaging the recent KTeV result with the older NA31 and E731 data, corresponding to $\varepsilon'/\varepsilon = (21.8 \pm 3) \times 10^{-4}$. The vertical lines show the ranges of the most recent published theoretical predictions, identified with the cities where most of the group members reside. The figure does not include two new results announced at this conference: on the experimental side, the first NA48 measurement [1] and, on the theoretical side, the new prediction based on the $1/N$ expansion [4], which I will refer to in the following as the Dortmund group estimate. The inclusion of the NA48 result $\varepsilon'/\varepsilon = (18.5 \pm 7.3) \times 10^{-4}$ lowers the experimental average shown in Fig. 1 by about 4%.

Looking at Fig. 1 two comments are in order. On the one hand, we should appreciate the fact that within the uncertainties of the theoretical calculations, there is indeed an overall agreement among the different predictions. All of them agree on the presence of a non-vanishing positive effect in the SM. On the other hand, the central values of the München (phenomenological $1/N$) and Rome (lattice) calculations are by a factor 3 to 5 lower than the averaged experimental central value.

In spite of the complexity of the calculations, I would like to emphasize that the difference between the predictions of the two estimates above and that of the Trieste group, based on the Chiral Quark Model (χ QM) [5],

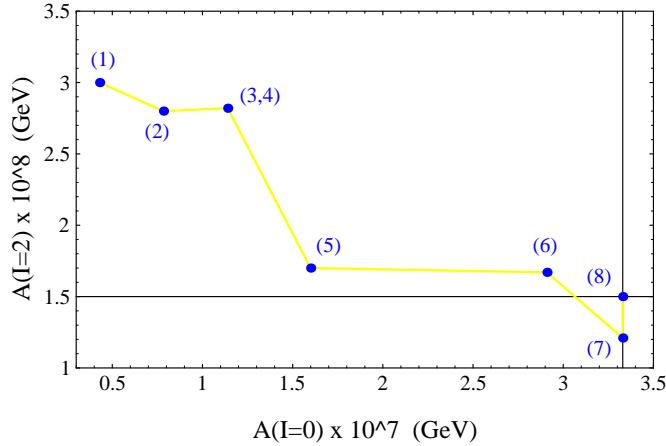


Figure 2: Anatomy of the $\Delta I = 1/2$ rule in the χ QM [8]. See the text for explanations. The cross-hairs indicate the experimental point.

is mainly due to the different size of the hadronic matrix element of the gluonic penguin Q_6 . In addition, I will show that the enhancement of the Q_6 matrix element in the χ QM approach can be simply understood in terms of chiral dynamics and, in this respect, it is related to the phenomenological embedding of the $\Delta I = 1/2$ selection rule.

The $\Delta I = 1/2$ selection rule in $K \rightarrow \pi\pi$ decays is known by some 40 years [6] and it states the fact that kaons are 400 times more likely to decay in the isospin zero two-pion state than in the isospin two component. This rule is not justified by any symmetry consideration and, although it is common understanding that its explanation must be rooted in the dynamics of strong interactions, there is no up to date derivation of this effect from first principle QCD.

As summarized by Martinelli at this conference [7] lattice cannot provide us at present with reliable calculations of the $I = 0$ penguin operators relevant to ε'/ε , as well as of the $I = 0$ components of the hadronic matrix elements of the tree-level current-current operators (penguin contractions), which are relevant for the $\Delta I = 1/2$ selection rule.

In the Munich approach the $\Delta I = 1/2$ rule is used in order to determine phenomenologically the matrix elements of $Q_{1,2}$ and, via operatorial relations, some of the matrix elements of the left-handed penguins. Unfortunately, the approach does not allow for a phenomenological determination of the matrix elements of the penguin operators which are most relevant for ε'/ε , namely the gluonic penguin Q_6 and the electroweak penguin Q_8 . Values in the ballpark of the leading $1/N$ estimate are assumed for these matrix elements, taking also into account that all present approaches show a suppression of $\langle Q_8 \rangle$ with respect to its vacuum saturation approximation (VSA).

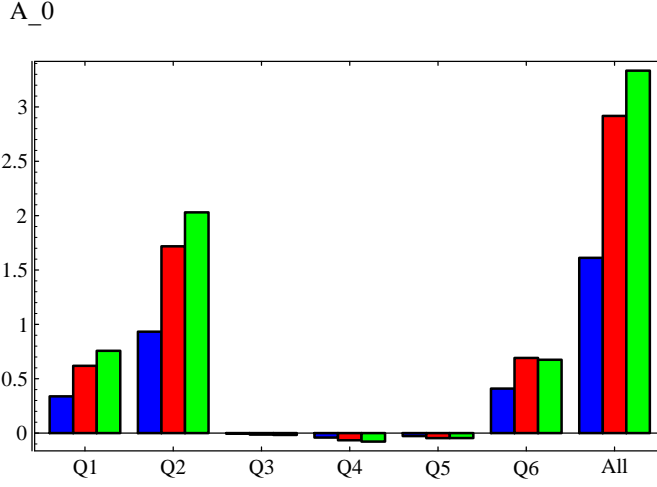


Figure 3: Anatomy of the $A(K^0 \rightarrow \pi\pi)_{I=0}$ amplitude (A_0) in units of 10^{-7} GeV for central values of the χ QM input parameters [8]: $O(p^2)$ calculation (black), with minimally subtracted chiral loops (half-tone), complete $O(p^4)$ result (light gray).

In the χ QM approach, the hadronic matrix elements can be computed as an expansion in momenta in terms of three parameters: the constituent quark mass, the quark condensate and the gluon condensate. The Trieste group has computed the $K \rightarrow \pi\pi$ matrix elements of the $\Delta S = 1, 2$ effective lagrangian up to $O(p^4/N)$ in the chiral and $1/N$ expansions [8].

Hadronic matrix elements and short distance Wilson coefficients are matched at a scale of 0.8 GeV as a reasonable compromise between the ranges of validity of perturbation theory and chiral lagrangian. By requiring the $\Delta I = 1/2$ rule to be reproduced within a 20% uncertainty one obtains a phenomenological determination of the three basic parameters of the model. This step is needed in order to make the model predictive, since there is no a-priori argument for the consistency of the matching procedure. As a matter of fact, all computed observables turn out to be very weakly scale dependent in a few hundred MeV range around the matching scale.

Fig. 2 shows an anatomy of the (model dependent) contributions which lead in the Trieste approach to reproducing the $\Delta I = 1/2$ selection rule.

Point (1) represents the result obtained by neglecting QCD and taking the factorized matrix element for the tree-level operator Q_2 , which is the only one present. The ratio A_0/A_2 is found equal to $\sqrt{2}$: a long way to the experimental point (8). Step (2) includes the effects of perturbative QCD renormalization on the operators $Q_{1,2}$ [9]. Step (3) shows the effect of including the gluonic penguin operators [10]. Electroweak penguins [11] are numerically negligible for the CP conserving amplitudes and are responsible for the very small shift in the A_2 direction. Therefore, perturbative QCD and factorization lead us from (1) to (4).

B_i	München 99 $\mu = 1.3 \text{ GeV}$	Roma 97 $\mu = 2.0 \text{ GeV}$	Trieste 97 $\mu = 0.8 \text{ GeV}$
$B_1^{(0)}$	13 (†)	—	9.5
$B_2^{(0)}$	6.1 (†)	—	2.9
$B_1^{(2)} = B_2^{(2)}$	0.48 (†)	—	0.41
B_3	1 (*)	1 (*)	-2.3
B_4	5.2 (* †)	$1 \div 6$ (*)	1.9
$B_5 \simeq B_6$	1.0 ± 0.3 (*)	1.0 ± 0.2	1.6 ± 0.3
$B_7^{(0)} \simeq B_8^{(0)}$	1 (*)	1 (*)	2.5 ± 0.1
$B_9^{(0)}$	7.0 (* †)	1 (*)	3.6
$B_{10}^{(0)}$	7.5 (* †)	1 (*)	4.4
$B_7^{(2)}$	1 (*)	0.6 ± 0.1	0.92 ± 0.02
$B_8^{(2)}$	0.8 ± 0.15 (*)	0.8 ± 0.15	0.92 ± 0.02
$B_9^{(2)}$	0.48 (†)	0.62 ± 0.10	0.41
$B_{10}^{(2)}$	0.48 (†)	1 (*)	0.41
\hat{B}_K	0.80 ± 0.15	0.75 ± 0.15	1.1 ± 0.2

Table 1: Summary of B factors. Legenda: (*) educated guess, (†) derived from the $\Delta I = 1/2$ rule. In the Trieste calculation the $\Delta I = 1/2$ rule is used to constrain the three basic model parameters in terms of which all matrix elements are computed.

Non-factorizable gluon-condensate corrections, a crucial model dependent effect, enter at the leading order in the chiral expansion leading to a substantial reduction of the A_2 amplitude (5), as first observed by Pich and de Rafael [12]. Moving the analysis to $O(p^4)$ the chiral loop corrections, computed on the LO chiral lagrangian via dimensional regularization and minimal subtraction, lead us from (5) to (6), while the corresponding $O(p^4)$ tree level counterterms calculated in the χ QM lead to the point (7). Finally, step (8) represents the inclusion of π - η - η' isospin breaking effects [13].

This model dependent anatomy shows the relevance of non-factorizable contributions and higher-order chiral corrections. The suggestion that chiral dynamics may be relevant to the understanding of the $\Delta I = 1/2$ selection rule goes back to the work of Bardeen, Buras and Gerard [14, 15] in the $1/N$ framework using a cutoff regularization. This approach has been recently revived and improved by the Dortmund group, with a particular attention to the matching procedure [4, 15]. A pattern similar to that shown in Fig. 2 for the chiral loop corrections to A_0 and A_2 was previously obtained in a NLO chiral lagrangian analysis, using dimensional regularization, by Missimer, Kambor and Wyler [16].

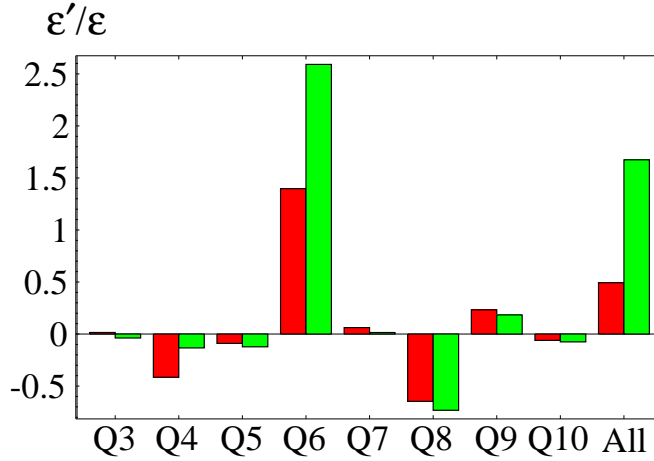


Figure 4: Predicting ε'/ε : a (Penguin) Comparative Anatomy of the München (dark gray) and Trieste (light gray) results (in units of 10^{-3}).

The χ QM approach allows us to further investigate the relevance of chiral corrections for each of the effective quark operators of the $\Delta S = 1$ lagrangian. Fig. 3 shows the contributions to the CP conserving amplitude A_0 of the relevant operators, providing us with a finer (model dependent) anatomy of the NLO chiral corrections. From Fig. 3 we notice that, because of the chiral loop enhancement, the Q_6 contribution to A_0 is about 20% of the total amplitude. As we shall see, the $O(p^4)$ enhancement of the Q_6 matrix element is what drives ε'/ε in the χ QM to the 10^{-3} ballpark.

A commonly used way of comparing the estimates of hadronic matrix elements in different approaches is via the so-called B factors which represent the ratio of the model matrix elements to the corresponding VSA values. However, care must be taken in the comparison of different models due to the scale dependence of the B 's and the values used by different groups for the parameters that enter the VSA expressions. Table 1 reports the B factors used for the predictions shown in Fig. 1.

An alternative pictorial and synthetic way of analyzing different outcomes for ε'/ε is shown in Fig. 4, where a “comparative anatomy” of the Trieste and München estimates is presented.

From the inspection of the various contributions it is apparent that the final difference on the central value of ε'/ε is almost entirely due to the difference in the Q_6 component. In second order, a larger (negative) contribution of the Q_4 penguin in the München calculation goes into the direction of making ε'/ε smaller.

The difference in the Q_4 contribution is easily understood. In the München estimate the Q_4 matrix element is obtained using the operatorial relation $\langle Q_4 \rangle = \langle Q_2 \rangle_0 - \langle Q_1 \rangle_0 + \langle Q_3 \rangle$, together with the knowledge acquired on $\langle Q_{1,2} \rangle_0$ from fitting the $\Delta I = 1/2$ selection rule at the charm

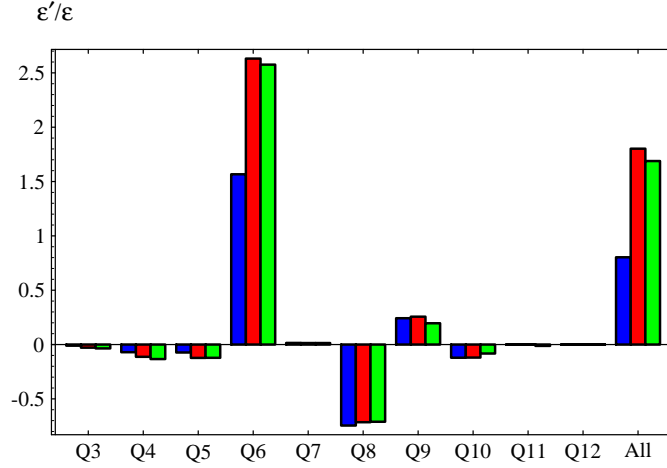


Figure 5: Anatomy of ε'/ε (in units of 10^{-3}) within the χ QM approach [8]. In black the LO results (which includes the non-factorizable gluonic corrections), in half-tone the effect of the inclusion of chiral-loop corrections and in light gray the complete $O(p^4)$ estimate.

scale.

As a matter of fact, the phenomenological fit of $\Delta I = 1/2$ rule requires a large value of $\langle Q_2 \rangle_0 - \langle Q_1 \rangle_0$ (which deviates by up to an order of magnitude from the naive VSA estimate). The assumption that $\langle Q_3 \rangle$ is given by its VSA value leads, in the München analysis, to a large value of $\langle Q_6 \rangle$: about 5 times larger than its VSA value. On the other hand, in the χ QM calculation $\langle Q_3 \rangle$ turns out to have a sign opposite to its VSA expression, in such a way that a smaller value for Q_4 is obtained. A lattice calculation of all gluonic penguins is definitely needed to disentangle such patterns.

At any rate, the main difference between the ε'/ε central values obtained in the Trieste and München calculations rests in the Q_6 matrix element. The nature of the difference is apparent in Fig. 5 where the various penguin contributions to ε'/ε in the Trieste analysis are further separated in LO (dark histograms) and NLO components—chiral loops (gray histograms) and $O(p^4)$ tree level counterterms (dark histograms).

It is clear that chiral loop dynamics plays a subleading role in the electroweak penguin sector (Q_{8-10}) while enhancing by 60% the Q_6 matrix element. At $O(p^2)$ the χ QM prediction for ε'/ε would just overlap with the München estimate once the small effect of the Q_4 operator is taken into account.

The χ QM analysis shows that the same dynamics that is relevant to the reproduction of the CP conserving A_0 amplitude (Fig. 3) is at work also in the CP violating sector (gluonic penguins).

In order to ascertain whether the model features represent real QCD effects one should wait for future improvements in lattice calculations [17].

On the other hand, indications for such a dynamics arise from current $1/N$ calculations [4, 18].

The idea of a connection between the $\Delta I = 1/2$ selection rule and ε'/ε is certainly not new [19, 20], although at the GeV scale, where one can trust perturbative QCD, penguins are far from providing the dominant contribution to the CP conserving amplitudes.

Before concluding, I like to make a comment on the role of the strange quark mass in the χ QM calculation of ε'/ε : in such an approach the basic parameter that enters the relevant penguin matrix elements is the quark condensate and the explicit dependence on m_s appears at the NLO in the chiral expansion. Varying the central value of $\bar{m}_s(m_c)$ from 150 MeV to 130 MeV affects $\langle Q_6 \rangle$ and $\langle Q_8 \rangle$ at the few percent level.

A more sensitive quantity is \hat{B}_K , which parametrizes the $\bar{K} - K$ matrix element. This parameter, which equals unity in the VSA turns out to be quite sensitive to $SU(3)$ breaking effects. Taking $\bar{m}_s(m_c) = 130 \pm 20$ MeV, $\Lambda_{\text{QCD}}^{(4)} = 340 \pm 40$ MeV and varying all relevant parameters, the updated χ QM result is:

$$\hat{B}_K = 1.0 \pm 0.2 ,$$

to be compared with the value used in the 1997 analysis (Table 1).

This increases the previous determination of $\text{Im } \lambda_t$ by roughly 10% and correspondingly ε'/ε (an updated analysis of ε'/ε in the χ QM with gaussian treatment of experimental inputs is in progress).

I conclude by summarizing the relevant remarks:

- Phenomenological approaches which embed the $\Delta I = 1/2$ selection rule in $K \rightarrow \pi\pi$ decays, generally agree with present lattice calculations in the pattern and size of the $I = 2$ components of the $\Delta S = 1$ hadronic matrix elements.
- Concerning the $I = 0$ matrix elements, where lattice calculations suffer from large systematic uncertainties, the $\Delta I = 1/2$ rule forces upon us large deviations from the naive VSA (see Table 1).
- In the Chiral Quark Model calculation, the fit of the CP conserving $K \rightarrow \pi\pi$ amplitudes, which determines the three basic parameters of the model, feeds down to the penguin sectors showing a substantial enhancement of the Q_6 matrix element, such that $B_6/B_8^{(2)} \approx 2$. This is what drives the ε'/ε prediction in the 10^{-3} ballpark.
- Up to 40% of the present uncertainty in the ε'/ε prediction arises from the uncertainty in the CKM elements $\text{Im}(V_{ts}^* V_{td})$ which is presently controlled by the $\Delta S = 2$ parameter B_K . A better determination of the unitarity triangle from B-physics is expected from the B-factories

and hadronic colliders. From K-physics, the rare decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ gives the cleanest “theoretical” determination of $\text{Im } \lambda_t$ [2].

- In spite of clever and interesting new-physics explanations for a large ε'/ε [21, 22, 23, 24], it is premature to interpret the present theoretical-experimental “disagreement” as a signal of physics beyond the SM. Ungauged systematic uncertainties affect presently all theoretical estimates. Not to forget the long-standing puzzle of the $\Delta I = 1/2$ rule: perhaps an “anomalously” large ε'/ε ($B_6/B_8^{(2)} \approx 2$) is just the CP violating projection of $A_0/A_2 \approx 20$.

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